Confidence Intervals for a Rate

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The Person Time module of Open Epi is used to analyze data where the numerator is a count of the events of interest and the denominator is the total person-time over which observations occurred. This method of analysis is frequently used in cohort studies and clinical trials. The idea is that a disease-free population is followed from a baseline. Person-time is the amount of time an individual accumulates until: 1) the study ends; 2) they develop the outcome of interest; or 3) they leave the study for some other reason. Person time is frequently expressed in person-years, although person-hours, days, or months will work just as well.

Single Person-Time Rate

For a single rate (also known as “incidence rate”), the numerator is the number of cases of the “disease,” and the denominator is the sum of person-years (or days, weeks, months) of exposure for all individuals prior to onset of the disease. The person-time variable represents the sum of the number of time units in which individuals were under study and disease-free. It should include units for those who never developed disease and those who were lost to follow-up after a defined period.

This module calculates various confidence intervals for a rate. First, the user is prompted to enter a numerator and denominator value:

![Confidence Intervals for a Rate](image)

The output from the example above is as follows:
The observed rate is 2 per 10 person-time units. Five different methods are used to calculate the confidence interval around this point estimate: Mid-P exact test, Fisher’s exact test, normal approximation, Byar approximation, and the Rothman/Greenland method. Of the five methods, the Mid-P exact test is generally the preferred method.

For confidence limit estimates < 0.0, the value 0.0 is shown. All confidence intervals calculated are two-sided and depend on the current setting of user’s choice (90%, 95%, 99%, 99.9% or 99.99%). Formulas for the methods are provided in the following section.

### Formulae

The notation for the formulae is:

- \( a \) = the observed numerator
- \( PT \) = is the observed denominator in person-time units
- \( rate = \frac{a}{PT} \)
- \( Z_{\frac{1}{2}} = \) the two-sided Z value (eg. \( Z=1.96 \) for a 95% confidence interval).

### Exact Tests (Mid-P and Fisher)

The limits for ‘a’ with 100(1-\( \alpha \)) percent confidence are the iterative solutions \( a \) and \( \bar{a} \).

Computing iterative solutions \( a \) and \( \bar{a} \) is below………

**Mid-P exact test (see Rothman and Boice):**
Lower bound: \( \left( \frac{1}{2} \right) e^{-\alpha} a^a a! + \sum_{k=0}^{a-1} \frac{e^{-\alpha} a^k}{k!} = 1 - \alpha / 2 \)

Upper bound: \( \left( \frac{1}{2} \right) e^{-\bar{a}} \bar{a}^a a! + \sum_{k=0}^{a-1} \frac{e^{-\bar{a}} \bar{a}^k}{k!} = \alpha / 2 \)

Fisher’s exact test (see Rothman and Boice):

Lower bound: \( \sum_{k=0}^{a} \frac{e^{-\alpha} a^k}{k!} = 1 - \alpha / 2 \)

Upper bound: \( \sum_{k=0}^{a} \frac{e^{-\bar{a}} \bar{a}^k}{k!} = \alpha / 2 \)

Therefore, the exact lower and upper limits for single person-time rate equal to “\( a/PT \)” would be \( \frac{a}{PT} \) and \( \frac{\bar{a}}{PT} \), respectively.

Normal Approximation:

\[
rate \pm Z_{1-\alpha/2} \sqrt{\frac{a}{PT^2}}
\]

Byar Method (see Rothman and Boice):

Lower bound: \( a \left( 1 - \frac{1}{9a} - \frac{Z_{1-\alpha/2}}{3} \sqrt{\frac{1}{a}} \right)^3 \)

Upper bound: \( (a+1) \left( 1 - \frac{1}{9(a+1)} + \frac{Z_{1-\alpha/2}}{3} \sqrt{\frac{1}{a+1}} \right)^3 \)
Rothman Greenland Method:

\[
\text{Lower bound: } e^{\ln(rate) - Z_{1-\alpha/2} \frac{1}{\sqrt{a}}} \\
\text{Upper bound: } e^{\ln(rate) + Z_{1-\alpha/2} \frac{1}{\sqrt{a}}}
\]

References


Update
The formulae for Mid-P and Fisher’s exact tests were added to the existing single person-time module on December 14, 2005.